

Heavy-light mesons spectrum from the nonperturbative QCD in the einbein field formalism

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Abstract

The spectrum of B and D mesons (including the low lying orbitally and radially excited states) is calculated using the quark-antiquark Hamiltonian derived from QCD in the einbein field formalism. Spin-spin and spin-orbit terms due to the confinement and OGE interactions are taken into account as perturbations. Results for the masses and splittings are confronted to the experimental and recent lattice data and are demonstrated to be in a reasonable agreement with both. We find that the orbital excitations with $l = 2$ and $l = 3$ for D meson lie approximately in the same region as its first radial excitation that might solve the mystery of the extremely narrow $D(2637)$ state recently claimed by DELPHI Collaboration.

1 Introduction

In spite of a rather long history of discussion and many theoretical attacks, an interest to the properties of heavy-light mesons is still very high. Indeed, heavy-light systems incorporate both properties of the light quarks which are extremely important for the physics of Chiral Symmetry Breaking (CSB), one of the most challenging phenomenon of the nonperturbative QCD, and heavy degrees of freedom which allow application of such profound methods of investigation as Heavy Quark Effective Theory (HQET) or Operator Product Expansion (OPE). The two main sources of information, experiment and lattice simulations, deliver new data on the heavy-light mesonic spectra, including orbital and radial excitations of the $q\bar{q}$ pair, which strongly need theoretical identification and description. In the present paper we calculate the masses of several low lying D and B mesonic states using the method developed in [1, 2] based on the Hamiltonian approach to the bound states problem in QCD. Starting with the quarks with current masses we intensively use the so-called einbein fields formalism [3, 4, 5] which not only considerably simplifies calculations, but allows to justify the treatment of the spin-spin and spin-orbit interactions as perturbations, naturally explaining the appearance of the “constituent” quark mass. Results for the spectra and splittings are compared with the experimental and lattice data and a good agreement is found.

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We demonstrate that in our model the proper account of the QCD string dynamics gives extra negative contribution into the energy of the orbital excitations and makes the orbitally excited state ($l = 2, n = 0$ and $l = 3, n = 0$) share the region of masses with the first radial excitation ($l = 0, n = 1$). The latter observation may resolve the problem of the resonance $D(2637)$ recently claimed by DELPHI Collaboration [6] and which is under intensive discussion at present (see *e.g.* [7]). Indeed, in spite of the fact that in our model both above mentioned states lie somewhat higher than the experimentally observed resonance, the latter can be identified with the orbital excitation of charm (with $l = 2$ or $l = 3$) rather than with the first radial excitation ($n = 1$) solving in such a way the problem of an extremely small width of the resonance inconsistent with any estimates of the $2S$ -state decays [7].

2 Einbein fields formalism and the mesonic Hamiltonian

The einbein fields formalism which has a long history in literature [3] was introduced as a method allowing to treat the kinematics of relativistic particles, but then it was generalized for the case of spinning particles and strings [8, 9]. One of its applications is the possibility to develop the canonical quantization procedure *a la* Dirac [10, 5]. In its simplest form this formalism can be applied to the point-like scalar relativistic particle described by the action

$$S = \int_{t_i}^{t_f} L dt \quad L = -m\sqrt{\dot{x}^2} \quad \dot{x}_\mu = \frac{\partial x_\mu}{\partial t} \quad (1)$$

so that the modified form of (1) looks like

$$L = -\frac{\mu\dot{x}^2}{2} - \frac{m^2}{2\mu}, \quad (2)$$

where the einbein field μ is introduced. Dynamics defined by the equations of motion for Lagrangians (1) and (2) is the same if the einbein field is treated as an independent degree of freedom and the corresponding constraints are introduced and properly treated. For a short review of the einbein fields formalism see *e.g.* [5] and references therein. In the mean time there exists another approach to the einbein fields [4] which allows to neglect their dependence on the proper time t and thus to treat them as variational parameters to be got rid of in the spectrum rather than in the Hamiltonian. Such an approach was used to a success in a set of papers (see *e.g.* [4, 11]) and proved to be rather accurate reproducing the relativistic spectra with the error within several per cent. In what follows we accept the above accuracy and treat einbeins variationally.

As it was shown in [2, 1] within the Vacuum Correlators Method (VCM) [12], writing the gauge invariant Euclidean Green's function of the $q\bar{q}$ meson

$$G_{q\bar{q}} = \Psi_{\bar{q}}^+(x_1)\Phi(x_1, x_2)\Psi_q(x_2), \quad (3)$$

with $\Phi(x_1, x_2)$ being the standard path-ordered parallel transporter operator

$$\Psi(x_1, x_2) = P \exp \left(ig \int_{x_2}^{x_1} dz_\mu A_\mu \right)$$

and employing Feynman–Schwinger representation for the single particle Green's functions and the area law for the isolated Wilson loop bounded by the quark and antiquark trajectories, one can arrive at the following result [2]

$$G_{q\bar{q}} = \int D\mu_1(t_1)D\mu_2(t_2)D\vec{x}_1D\vec{x}_2 e^{-K_1-K_2} Tr \left[\Gamma^{(f)}(m_1 - \hat{D})\Gamma^{(i)}(m_2 - \hat{\bar{D}}) \times \right. \quad (4)$$

$$P_\sigma \exp \left(\int_0^T \frac{dt_1}{2\mu_1(t_1)} \sigma_{\mu\nu}^{(1)} \frac{\delta}{i\delta s_{\mu\nu}(x_1(t_1))} \right) \exp \left(- \int_0^T \frac{dt_2}{2\mu_2(t_2)} \sigma_{\mu\nu}^{(2)} \frac{\delta}{i\delta s_{\mu\nu}(x_2(t_2))} \right) \exp(-\sigma S) \Big],$$

where μ_1 and μ_2 are the einbein fields, K_i being the kinetic energies of the quarks

$$K_i = \int_0^T dt_i \left(\frac{m_i^2}{2\mu_i} + \frac{\mu_i}{2} + \frac{\mu_i \dot{x}_i^2}{2} \right), \quad i = 1, 2, \quad (5)$$

S stands for the minimal area swept by the quark-antiquark trajectories, $\sigma_{\mu\nu} = \frac{1}{4i}(\gamma_\mu \gamma_\nu - \gamma_\nu \gamma_\mu)$ and $\delta/\delta s_{\mu\nu}$ denotes the derivative with respect to the element of the area S . The form of the kinetic energy (5) immediately follows from the equation (2) if the laboratory gauge $t = x_0$ is fixed in the proper time reparametrization group. Equation (4) allows to find the Hamiltonian of the quark-antiquark pair connected with the relativistic string. If one neglects spin-dependent terms in (4), then the corresponding Hamiltonian coincides by the one for the Nambu-Goto string with scalar quarks at the ends which follows from the Lagrangian

$$L(t) = -m_1 \sqrt{\dot{x}_1^2} - m_2 \sqrt{\dot{x}_2^2} - \sigma \int_0^1 d\beta \sqrt{(\dot{w}w')^2 - \dot{w}^2 w'^2} \quad x_{10} = x_{20} = t, \quad (6)$$

where the minimal string is parametrized by the profile function $w(t, \beta)$, the dot and the prime denote its derivatives with respect to the proper time t and the string coordinate β respectively. Thus for the centre-of-mass frame one arrives at [1]

$$H = \sum_{i=1}^2 \left(\frac{p_r^2 + m_i^2}{2\mu_i} + \frac{\mu_i}{2} \right) + \int_0^1 d\beta \left(\frac{\sigma^2 r^2}{2\nu} + \frac{\nu}{2} \right) + \frac{\vec{L}^2}{2r^2[\mu_1(1-\zeta)^2 + \mu_2\zeta^2 + \int_0^1 d\beta \nu(\beta - \zeta)^2]},$$

$$\zeta = \frac{\mu_1 + \int_0^1 d\beta \nu \beta}{\mu_1 + \mu_2 + \int_0^1 d\beta \nu}. \quad (7)$$

Note that an extra einbein ν following equation (7) has the meaning of the string energy density. Extrema in all three einbeins (μ_1 , μ_2 and ν) should be taken in (7). For the case of zero angular momentum $L = 0$ the last term in (7) vanishes and the extremal value for ν is easily found to be

$$\nu_0 = \sigma r, \quad (8)$$

so that the resulting interaction gives the linearly rising potential σr . Expansion of the last term in Hamiltonian (7) in powers of $\sqrt{\sigma}/\mu$ with the substitution of the extremal value ν_0 yields

$$H = H_0 + V_{string} \quad (9)$$

$$H_0 = \sum_{i=1}^2 \left(\frac{\vec{p}^2 + m_i^2}{2\mu_i} + \frac{\mu_i}{2} \right) + \sigma r - \frac{4}{3} \frac{\alpha_s}{r} - C_0, \quad (10)$$

$$V_{string} \approx - \frac{\sigma(\mu_1^2 + \mu_2^2 - \mu_1\mu_2)}{6\mu_1^2\mu_2^2} \frac{\vec{L}^2}{r} \quad (11)$$

where we have supplied H_0 with the constant term C_0 and the colour Coulomb interaction which comes from the perturbative gluon exchange omitted in (4) for simplicity. Equation (11) gives the first spin-independent correction called the string correction [13, 1] which is known to reduce the inverse Regge trajectory slope $dM_L^2/dL = 4\sigma$ specific for the heavy-light limit of the Hamiltonian H_0 , and, when taken at the account at full scale, to lead to the correct string slope $\pi\sigma$ [1, 11].

In what follows we shall consider H_0 as the zeroth approximation for calculation of the mesonic spectrum. Once the angular momentum \vec{L} , the total spin of the quark-antiquark pair \vec{S} and the total momentum $\vec{J} = \vec{L} + \vec{S}$ are separately conserved with the Hamiltonian H_0 then one can specify the mesonic states as terms $n^{2S+1}L_J$, where n is the radial excitation number.

Let us turn back to the equation (4) and to identify the spin-dependent correction to the zeroth order Hamiltonian H_0 . Following [2] one finds

$$V_{sd} = \frac{8\pi\kappa}{3\mu_1\mu_2}(\vec{S}_1\vec{S}_2)|\psi(0)|^2 - \frac{\sigma}{2r}\left(\frac{\vec{S}_1\vec{L}}{\mu_1^2} + \frac{\vec{S}_2\vec{L}}{\mu_2^2}\right) + \frac{\kappa}{r^3}\left(\frac{1}{\mu_1} + \frac{1}{\mu_2}\right)\left(\frac{\vec{S}_1\vec{L}}{\mu_1} + \frac{\vec{S}_2\vec{L}}{\mu_2}\right) \quad (12)$$

$$+ \frac{\kappa}{\mu_1\mu_2r^3}\left(3(\vec{S}_1\vec{n})(\vec{S}_2\vec{n}) - (\vec{S}_1\vec{S}_2)\right) + \frac{\kappa^2}{2\pi\mu^2r^3}(\vec{S}\vec{L})(2 - \ln(\mu r) - \gamma_E), \quad \gamma_E = 0.57$$

where $\kappa = \frac{4}{3}\alpha_s$ and we have added the term of order α_s^2 which comes from one-loop calculations and is extensively discussed in literature [14, 15]. Note that up to the last term potential (12) coincides in form with the Eichten–Feinberg–Gromes results [16], but contains μ 's in the denominators instead of masses. In the meantime as clearly seen from (10) $\mu_i \sim \sqrt{\langle \vec{p}^2 \rangle + m_i^2} > m_i$ or even $\mu_i \gg m_i$ for light quarks¹ (see also Table 1). Einbeins μ 's can be viewed as dynamical or “constituent” quark masses that makes it possible to consider (12) as a correction and to justify the expansion made in (11) (see *e.g.* [17] for more detailed discussion of this issue).

3 Numerical solution

3.1 Spectrum of the Hamiltonian H_0

As mentioned in the introduction einbein fields can considerably simplify the relativistic dynamics. Indeed with the einbeins μ 's introduced the kinetic part of the Hamiltonian H_0 from (10) has a nonrelativistic form, so that the corresponding Schrödinger equation can be written in the reduced dimensionless form (see *e.g.* [18, 17])

$$\left(-\frac{d^2}{d\vec{x}^2} + |\vec{x}| - \frac{\lambda}{|\vec{x}|}\right)\chi_\lambda = a(\lambda)\chi_\lambda \quad \lambda = \kappa\left(\frac{2\mu}{\sqrt{\sigma}}\right)^{2/3} \quad (13)$$

On finding the solutions of equation (13) for χ_λ and $a(\lambda)$ as functions of the reduced Coulomb interaction strength λ one has for the extremal values of the einbeins:

$$\mu_1(\lambda) = \sqrt{m_1^2 + \Delta^2(\lambda)} \quad \mu_2(\lambda) = \sqrt{m_2^2 + \Delta^2(\lambda)} \quad \mu(\lambda) = \frac{1}{2}\sqrt{\sigma}\left(\frac{\lambda}{\kappa}\right)^{3/2}, \quad (14)$$

where

$$\Delta^2(\lambda) = \frac{\sigma\lambda}{3\kappa}\left(a + 2\lambda\left|\frac{\partial a}{\partial \lambda}\right|\right).$$

Then the definition of the reduced field μ gives the equation for λ

$$\mu(\lambda) = \frac{\mu_1(\lambda)\mu_2(\lambda)}{\mu_1(\lambda) + \mu_2(\lambda)}, \quad (15)$$

¹Let us remind here that these are current quark masses to be denoted m_i and to enter equations (1)-(11)

n	l	meson	m_1	m_2	σ	α_s	λ	μ_1	μ_2	μ	E_0	$ \psi(0) $
0	0	D	1.4	0.009	0.17	0.4	0.817	1.497	0.529	0.391	2.198	0.161
		D_s	1.4	0.17	0.17	0.4	0.847	1.501	0.569	0.412	2.224	0.167
		B	4.8	0.005	0.17	0.39	0.999	4.840	0.619	0.549	5.527	0.209
		B_s	4.8	0.17	0.17	0.39	1.035	4.842	0.658	0.579	5.550	0.219
0	1	D	1.4	0.009	0.17	0.4	0.869	1.522	0.597	0.428	2.640	0
		D_s	1.4	0.17	0.17	0.4	0.891	1.525	0.629	0.445	2.663	0
		B	4.8	0.005	0.17	0.39	1.052	4.847	0.675	0.593	5.949	0
		B_s	4.8	0.17	0.17	0.39	1.080	4.849	0.707	0.617	5.970	0
0	2	D	1.4	0.009	0.17	0.4	0.924	1.554	0.674	0.470	2.961	0
		D_s	1.4	0.17	0.17	0.4	0.942	1.557	0.702	0.484	2.982	0
		B	4.8	0.005	0.17	0.39	1.128	4.860	0.762	0.659	6.245	0
		B_s	4.8	0.17	0.17	0.39	1.151	4.861	0.789	0.679	6.263	0
1	0	D	1.4	0.009	0.17	0.4	0.929	1.557	0.682	0.474	2.848	0.162
		D_s	1.4	0.17	0.17	0.4	0.947	1.561	0.710	0.488	2.869	0.165
		B	4.8	0.005	0.17	0.39	1.142	4.863	0.779	0.671	6.131	0.207
		B_s	4.8	0.17	0.17	0.39	1.165	4.864	0.806	0.692	6.149	0.212

Table 1: Solutions of the equations (13)-(15) for standard values of the string tension σ , the strong coupling constant α_s and the masses of the quarks. All parameters are given in GeV to the appropriate powers.

which is the subject to numerical investigation. Unfortunately there is no analytic solution for $a(\lambda)$, so one has to generate a selfconsistent solution of both equations, (13) and (15).

Let us give here two more formulae which will be needed for calculating the spin-spin and spin-orbit splittings. For radially excited states one can find [18, 17]

$$|\psi(0)|^2 = \frac{2\mu\sigma}{4\pi} \left(1 + \lambda \langle r^{-2} \rangle\right), \quad (16)$$

where

$$\langle r^N \rangle = (2\mu\sigma)^{N/3} \int_0^\infty x^{N+2} |\chi_\lambda(x)|^2 dx, \quad N > -3 - 2l. \quad (17)$$

In Table 1 we give the results of numerical calculations with the Hamiltonian H_0 for D , D_s , B and B_s mesons.

3.2 Spin-spin and spin-orbit splittings. Comparison with the experimental and lattice data

In this subsection we calculate the contribution of the V_{string} and V_{sd} terms. First of all we collect the averaged values of the spin-orbit and spin-tensor interactions between various $1P$ and $1D$ eigenstates of the Hamiltonian H_0

$$\begin{aligned}
\langle {}^1P_1 | \vec{S}_1 \vec{L} | {}^1P_1 \rangle &= 0 & \langle {}^1P_1 | \vec{S}_2 \vec{L} | {}^1P_1 \rangle &= 0 & \langle {}^1P_1 | (\vec{S}_1 \vec{n})(\vec{S}_2 \vec{n}) | {}^1P_1 \rangle &= -\frac{1}{4} \\
\langle {}^3P_0 | \vec{S}_1 \vec{L} | {}^3P_0 \rangle &= -1 & \langle {}^3P_0 | \vec{S}_2 \vec{L} | {}^3P_0 \rangle &= -1 & \langle {}^3P_0 | (\vec{S}_1 \vec{n})(\vec{S}_2 \vec{n}) | {}^3P_0 \rangle &= -\frac{1}{4} \\
\langle {}^3P_1 | \vec{S}_1 \vec{L} | {}^3P_1 \rangle &= -\frac{1}{2} & \langle {}^3P_1 | \vec{S}_2 \vec{L} | {}^3P_1 \rangle &= -\frac{1}{2} & \langle {}^3P_1 | (\vec{S}_1 \vec{n})(\vec{S}_2 \vec{n}) | {}^3P_1 \rangle &= \frac{1}{4} \\
\langle {}^3P_2 | \vec{S}_1 \vec{L} | {}^3P_2 \rangle &= \frac{1}{2} & \langle {}^3P_2 | \vec{S}_2 \vec{L} | {}^3P_2 \rangle &= \frac{1}{2} & \langle {}^3P_2 | (\vec{S}_1 \vec{n})(\vec{S}_2 \vec{n}) | {}^3P_2 \rangle &= \frac{1}{20}
\end{aligned} \quad (18)$$

$$\begin{aligned}
\langle {}^1D_2 | \vec{S}_1 \vec{L} | {}^1D_2 \rangle &= 0 & \langle {}^1D_2 | \vec{S}_2 \vec{L} | {}^1D_2 \rangle &= 0 & \langle {}^1D_2 | (\vec{S}_1 \vec{n})(\vec{S}_2 \vec{n}) | {}^1D_2 \rangle &= -\frac{1}{4} \\
\langle {}^3D_1 | \vec{S}_1 \vec{L} | {}^3D_1 \rangle &= -\frac{3}{2} & \langle {}^3D_1 | \vec{S}_2 \vec{L} | {}^3D_1 \rangle &= -\frac{3}{2} & \langle {}^3D_1 | (\vec{S}_1 \vec{n})(\vec{S}_2 \vec{n}) | {}^3D_1 \rangle &= -\frac{1}{12} \\
\langle {}^3D_2 | \vec{S}_1 \vec{L} | {}^3D_2 \rangle &= -\frac{1}{2} & \langle {}^3D_2 | \vec{S}_2 \vec{L} | {}^3D_2 \rangle &= -\frac{1}{2} & \langle {}^3D_2 | (\vec{S}_1 \vec{n})(\vec{S}_2 \vec{n}) | {}^3D_2 \rangle &= \frac{1}{4} \\
\langle {}^3D_3 | \vec{S}_1 \vec{L} | {}^3D_3 \rangle &= 1 & \langle {}^3D_3 | \vec{S}_2 \vec{L} | {}^3D_3 \rangle &= 1 & \langle {}^3D_3 | (\vec{S}_1 \vec{n})(\vec{S}_2 \vec{n}) | {}^3D_3 \rangle &= \frac{1}{28}
\end{aligned} \tag{19}$$

as well as the transition matrix elements

$$\begin{aligned}
\langle {}^1P_1 | \vec{S}_1 \vec{L} | {}^3P_1 \rangle &= \frac{1}{\sqrt{2}} & \langle {}^1P_1 | \vec{S}_2 \vec{L} | {}^3P_1 \rangle &= -\frac{1}{\sqrt{2}} \\
\langle {}^1D_2 | \vec{S}_1 \vec{L} | {}^3D_2 \rangle &= \sqrt{\frac{3}{2}} & \langle {}^1D_2 | \vec{S}_2 \vec{L} | {}^3D_2 \rangle &= -\sqrt{\frac{3}{2}},
\end{aligned} \tag{20}$$

which lead to mixing of $|{}^1P_1\rangle$, $|{}^3P_1\rangle$, and $|{}^1D_2\rangle$, $|{}^3D_2\rangle$ pairs so that the physical state is subject to the matrix equations of the following type:

$$\begin{vmatrix} E_1 - E & V_{12} \\ V_{12}^* & E_2 - E \end{vmatrix} = 0 \tag{21}$$

The results of our numerical calculations as well as the experimental and lattice data are given in Tables 2,3. The only fitting parameter we use here is the constant C_0 which takes the following values:

$$C_0(D) = 212 MeV \quad C_0(D_s) = 124 MeV \quad C_0(B) = 203 MeV \quad C_0(B_s) = 124 MeV. \tag{22}$$

As clearly seen from (22) the constant C_0 practically does not depend on the heavy quark and is completely defined by the properties of the light one.

4 Discussion and conclusions

Several comments concerning our choice of parameters are in order here. From Table 1 it is seen that we use practically the same values of the strong coupling constant α_s for all states. To justify this action let us note that in the case of a light particle moving in the field of a heavy one the OGE interaction between them depends not on the total mass of the system, but rather on its size. Then the slight variation of the strong coupling is due to small differences in the mesons sizes. The exact value of α_s is chosen to be near its frozen value [19] that emphasizes the nonperturbative nature of the processes responsible for the formation of the heavy-light mesons spectrum. The fits are very weakly sensible to variations of the heavy quarks mass which lead mainly to rescaling of the overall shift constant C_0 , so that the latter remains practically the only fitting parameter as stated in the preceding section.

In conclusion we would like to comment on the possible identification of the resonance $D(2637)$ recently claimed by DELPHI Collaboration [6]. It was reported to be extremely narrow, about $15 MeV$, that leads to a confusion as such a small width is inconsistent with the identification of this state as the first radial excitation $D^{*'}(J^P = 1^-)$. It was found in a number of papers [7] that in spite of the fact that the mass of this state perfectly coincides with the predictions of quark models, all estimates of its width fail to give such a small value. In the meantime it was found that widths of orbitally excited D mesons with quantum

Meson	$n^{2S+1}L_J$	J^P	M_{exp}	M_{theor}	M_{lat}
D	1^1S_0	0^-	1869	1876	1884
D^*	1^3S_1	1^-	2010	2022	1994
D_1	$1^1P_1/{}^3P_1$	1^+	2420	2354 <u>2403</u>	
D_2	1^3P_2	2^+	2460	2432	
$D^{*'} $	1^3D_3	3^-	2637	<u>2654</u>	
	$1^1D_2/{}^3D_2$	2^-		<u>2663</u>	
				2729	
	2^3S_1	0^-		<u>2664</u>	
D_s	1^1S_0	0^-	1968	1990	1984
D_s^*	1^3S_1	1^-	2112	2137	2087
D_{1s}	$1^1P_1/{}^3P_1$	1^+	2536	2471 <u>2516</u>	2494
D_{2s}	1^3P_2	2^+	2573	2547	2411
B	1^1S_0	0^-	5279	5277	5293
B^*	1^3S_1	1^-	5325	5340	5322
B_1	$1^1P_1/{}^3P_1$	1^+	5732	5685 <u>5719</u>	
B_2	1^3P_2	2^+	5731	5820	
$B^{*'} $	1^3D_3	3^-	5860	<u>5955</u>	
	$1^1D_2/{}^3D_2$	2^-		<u>5953</u>	
				6018	
	2^3S_1	0^-		<u>5940</u>	5890
B_s	1^1S_0	0^-	5369	5377	5383
B_s^*	1^3S_1	1^-	5416	5442	5401
B_{1s}	$1^1P_1/{}^3P_1$	1^+	5853	5789 <u>5819</u>	5783
B_{2s}	1^3P_2	2^+		5834	5848

Table 2: Masses of the D , D_s , B and B_s mesons in MeV . Lattice results are extracted from Figures 26,27 and Tables XXVIII,XXIX of [20]. Symbols $1^1P_1/{}^3P_1$ and $1^1D_2/{}^3D_2$ are used to indicate that the physical states are mixtures of the 1^1P_1 and 3P_1 or 1^1D_2 and 3D_2 states correspondingly. Underlined figures give masses of the most probable candidates for the experimentally observed resonances.

Splitting	$D_s - D$	$D_s^* - D^*$	$D^* - D$	$D_s^* - D_s$	$B_s - B$	$B_s^* - B^*$	$B^* - B$	$B_s^* - B_s$
Experim.	99	102	141	144	90	91	46	47
Theory	114	115	146	147	100	102	63	65
Lattice	100	92	110	103	90	90	30	29

Table 3: Splittings for the D , D_s , B and B_s mesons in MeV . Lattice results are taken from Tables XXVIII,XXIX of [20].

numbers 2^- , 3^- could be consistent with the reported value [7], but then the following two points were put forward as main objections: i) a neighboring slightly more massive state should be observed as well, ii) quark models predict orbitally excited mesons to be at least 50MeV heavier than needed. Here we would like to comment on the second argument. As one can find from the fit given in Table 2, in our model the first radial and the second orbital excitations share the same region of masses. It is not surprise as the negative mass shift of about 50MeV for the orbitally excited state, missing in standard quark models, is readily delivered by the string correction given by equation (11). This contribution does not affect radially excited states but is significant for orbital excitations. It was demonstrated in [11] that the account of the proper dynamics of the QCD string in mesons brings the slope of Regge trajectories to their correct values. Appearance of the term (11) in the meson Hamiltonian and its important contribution into the masses of orbitally excited states is yet another reflection of the general situation that the proper QCD string dynamics is extremely important in description of hadronic properties and thus it should be taken into account. So we conclude that the mysterious $D(2637)$ state indeed can be identified with the orbital excitation 2^- or 3^- rather than with the radial one 0^- , that resolves the second objective above. This statement may hold true for the corresponding states in the B meson spectrum, where DELPHI Collaboration also claims a similar state [21].

As far as the first one is concerned, our model also predicts another orbitally excited state with the mass 2728MeV , *i.e.* 65MeV higher than D'_2 given in the table, and its experimental grounds are really not clear.

Our predictions for $D(2654)$, $D(2663)$ and $D(2664)$ lie somewhat higher than the experimentally observed value. We find it to be a reflection of a general lack of the “ μ -technique” used in this paper, which gives larger errors for higher excited states. The difference of about $15 - 20\text{MeV}$ between the theoretical predictions and the experimental datum $2637 \pm 6\text{MeV}$ can be explained in this way. Development of a systematical approach to the einbein fields as variational parameters could shed light on the sources of systematical errors and possibly to improve the results presented in this paper.

The authors would like to thank Yu.A.Simonov for constant interest to the work and useful discussions, and B.L.G.Bakker for providing the numerical solutions for the reduced Schrödinger equation. Financial support of RFFI grants 00-02-17836 and 00-15-96786 and INTAS-RFFI grant IR-97-232 is gratefully acknowledged.

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